Sequences and Series 1 - Solutions

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1. (a) Arithmetic series $a = 4$ $d = 6$ $u_n = 4 + (n-1).6 = 334$

$n = 56$ ✓

$$S_{56} = \frac{56}{2}(2\times 4 + 55\times 6) = 9464$$ ✓

(b) $\frac{21}{4}, \frac{18}{4}, \frac{15}{4}, \ldots$ $a = \frac{21}{4}$ $d = \frac{-3}{4}$ $u_n = \frac{21}{4} + \frac{-3}{4}(n-1) = -3$

$\frac{-3}{4}(n-1) = \frac{-33}{4}$

$n = 12$ ✓

$$S_{12} = \frac{12}{2}\left(2\times\frac{21}{4} + 11\times\frac{-3}{4}\right) = \frac{27}{2}$$ ✓

(c) Geometric series $a = 100$ $r = 1.1$ $n - 1 = 10$

$n = 11$ ✓

$$S_{11} = \frac{100(1.1^{11} - 1)}{1.1 - 1} \approx 1853.1$$ ✓

2. $d = -3$ ✓

$u_9 = a - 3\times 8 = 15$ ✓

$a = 39$

$$S_{10} = \frac{10}{2}(2\times 39 + 9\times(-3)) = 255$$ ✓

3. $u_3 = a + 2d = 17$

$u_7 = a + 6d = 33$ ✓

$4d = 16$

$d = 4$ ✓

$a = 9$

$$S_{20} = \frac{20}{2}(2\times 9 + 19\times 4) = 940$$ ✓

4 (a) $S_n = \frac{n}{2}\left(4 + 3(n-1)\right)$

$= \frac{n}{2}(4 + 3n - 3)$

$= \frac{n}{2} + \frac{3}{2}n^2 = 610$ ✓

$3n^2 + n - 1220 = 0$ ✓

$(3n + 61)(n - 20) = 0$ ✓

$n \in \mathbb{N}$ so $n = 20$ ✓

(b) $S_n = \frac{n}{2} + \frac{3}{2}n^2 > 1000$ ✓

$3n^2 + n - 2000 > 0$ ✓

Solving $3n^2 + n - 2000 = 0$

$n = \frac{-1 \pm \sqrt{1^2 + 4 \times 3 \times 2000}}{6}$

$= \frac{-1 \pm \sqrt{24001}}{6}$

$n = 25.65$ or $n = -25.99$ ✓

The least value of $n \in \mathbb{N}$ for which $3n^2 + n - 2000 > 0$ is $n = 26$ ✓

5. (a) $r = \frac{-42}{84} = -\frac{1}{2}$ ✓ $|r| < 1$ so S_∞ exists. ✓

$S_\infty = \frac{84}{1 + \frac{1}{2}} = \frac{84 \times 2}{3} = 56$ ✓

(b) $r = -\frac{1}{9}$ ✓ $|r| < 1$ so S_∞ exists. ✓

$S_\infty = \frac{1}{1 + \frac{1}{9}} = \frac{9}{10}$ ✓

(c) $r = \frac{16}{64} = \frac{1}{4}$ ✓ $|r| < 1$ so S_∞ exists. ✓

$S_\infty = \frac{64}{1-1/4} = \frac{64 \times 4}{3} = \frac{256}{3}$ ✓

6. $u_3 = 60 \times r^2 = 15$ ✓

$r^2 = \frac{15}{60} = \frac{1}{4}$

$r = \pm\sqrt{\frac{1}{4}}$

$= \pm \frac{1}{2}$ ✓

$S_\infty = \frac{60}{1-1/2}$ or $S_\infty = \frac{60}{1+1/2}$

$= 120$ ✓ $= 40.$ ✓