6A

1(a) $\underline{F}_1 + \underline{F}_2 + \underline{F}_3 = \underline{0}$

$$\begin{pmatrix} 6 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ -5 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$4 + a = 0$

$\underline{\underline{a = -4}}$

$-1 + b = 0$

$\underline{\underline{b = 1}}$

(b) $$\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -5 \\ 2 \end{pmatrix} + \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$5 + a = 0$

$\underline{\underline{a = -5}}$

$-2 + b = 0$

$\underline{\underline{b = 2}}$

$-1 + 2 + c = 0$

$\underline{\underline{c = -1}}$

(c) $$\begin{pmatrix} 5 \\ a \\ 1 \end{pmatrix} + \begin{pmatrix} b \\ -6 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$a - 4 = 0$ $\quad$ $b + 2 = 0$ $\quad$ $\underline{\underline{c = 0}}$

$\underline{\underline{a = 4}}$ $\quad$ $\underline{\underline{b = -2}}$

6A

(2) $\begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}2\sqrt{3}\cos\theta\\2\sqrt{3}\sin\theta\end{pmatrix} + \begin{pmatrix}0\\x\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$ as in equilibrium

$$3 + 2\sqrt{3}\cos\theta = 0$$

$$2\sqrt{3}\cos\theta = -3$$

$$\cos\theta = \frac{-3}{2\sqrt{3}}$$

$$\left(\frac{3}{2\sqrt{3}} = \frac{\sqrt{9}}{2\sqrt{3}} = \frac{\sqrt{3}}{2}\right)$$

$$= -\frac{\sqrt{3}}{2}$$

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= 30^\circ$$

$$\theta = 180 - 30,\ 180 + 30$$

$$\theta = 150^\circ,\ 210^\circ$$

$$\theta = 150^\circ$$

assume θ is obtuse.

$$2\sqrt{3}\sin\theta + x = 0$$

$$2\sqrt{3}\sin(150) + x = 0$$

$$2\sqrt{3}\left(\frac{1}{2}\right) + x = 0$$

$$\sqrt{3} + x = 0$$

$$x = -\sqrt{3}$$

6A

(3) $\begin{pmatrix} 87 \\ -35 \end{pmatrix} + \begin{pmatrix} -148 \\ -71 \end{pmatrix} + \begin{pmatrix} 61 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 106 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

resultant force of $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ means the sledge is in equilibrium, i.e. not moving.

(4) $\underline{F}_1 + \underline{F}_2 + \underline{F}_3 = \underline{0}$ to keep the dogs still.

$\begin{pmatrix} 40 \\ 0 \end{pmatrix} + \begin{pmatrix} -25 \\ 37 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$15 + x = 0$

$\underline{x = -15}$

$37 + y = 0$

$\underline{y = -37}$

$\underline{\underline{F}_3 = \begin{pmatrix} -15 \\ -37 \end{pmatrix}}$

(5)(b) $|\underline{F}_3| = \sqrt{(-15)^2 + (-37)^2}$

$= 39.924\ldots$

$= \underline{40N}$

6A

(5) $\begin{pmatrix}0\\-5\end{pmatrix}+\begin{pmatrix}0\\7\end{pmatrix}+\begin{pmatrix}15\\0\end{pmatrix}+\begin{pmatrix}t_1\\t_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ as stationary.

$15+t_1=0$

$t_1=-15$

$2+t_2=0$

$t_2=-2$ $\qquad \underline{t}=\begin{pmatrix}-15\\-2\end{pmatrix}$

$|\underline{t}|=\sqrt{(-15)^2+(-2)^2}$

$=15.1\,N$

(6) $\begin{pmatrix}7.6\\0\end{pmatrix}+\begin{pmatrix}0\\40.9\end{pmatrix}+\begin{pmatrix}-20\\-34.6\end{pmatrix}+\begin{pmatrix}q_1\\q_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

~~-12~~. $-12.4+q_1=0$

$q_1=12.4$

$6.3+q_2=0$

$q_2=-6.3$

$\underline{q}=\begin{pmatrix}12.4\\-6.3\end{pmatrix}$

$|\underline{q}|=\sqrt{(12.4)^2+(-6.3)^2}$

$|\underline{q}|=13.908$

$=14N$

6A

7 (a) $\begin{pmatrix} 0 \\ 0 \\ -980 \end{pmatrix} + \begin{pmatrix} -314 \\ -235 \\ 230 \end{pmatrix} + \begin{pmatrix} 784 \\ 0 \\ 460 \end{pmatrix} + \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$470 + F_1 = 0$

$F_1 = -470$

$-235 + F_2 = 0$

$F_2 = 235$

$-290 + F_3 = 0$

$F_3 = 290$

a $\underline{F}_A = \begin{pmatrix} -470 \\ 235 \\ 290 \end{pmatrix} N$

(b) $|\underline{F}_A|$

$= \sqrt{(-470)^2 + (235)^2 + (290)^2}$

$= 600.187..$

$= \underline{\underline{600 N}}$

6A

(8) $$\begin{pmatrix} 0 \\ 0 \\ 800 \end{pmatrix} + \begin{pmatrix} -120 \\ 0 \\ -160 \end{pmatrix} + \begin{pmatrix} 0 \\ 210 \\ -360 \end{pmatrix} + \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$x - 120 = 0$$
$$x = 120$$

$$210 + y = 0$$
$$y = -210$$

$$280 + z = 0$$
$$z = -280$$

$$\underline{F}_B = \begin{pmatrix} 120 \\ -210 \\ -280 \end{pmatrix}$$

$$|\underline{F}_B| = \sqrt{120^2 + (-210)^2 + (-280)^2}$$
$$= \underline{\underline{370\,N}}$$

6B

1 (a)

$\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}|\cos\theta$

$= 3 \times 2 \times \cos 60$

$= 3 \times 2 \times \frac{1}{2}$

$= \underline{\underline{3}}$

(b) $\underline{a} \cdot \underline{b}$

$= |\underline{a}||\underline{b}|\cos\theta$

$= 1 \times 2 \times \cos \pi/4$

$= 1 \times 2 \times \frac{1}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}}$

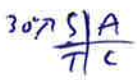
$= \frac{\sqrt{4}}{\sqrt{2}}$

$= \underline{\underline{\sqrt{2}}}$

(c) $\underline{a} \cdot \underline{b}$

$= |\underline{a}||\underline{b}|\cos\theta$

$= 5 \times 3 \times \cos 150$

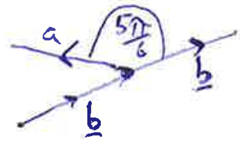


$= 5 \times 3 \times -\cos 30$

$= 5 \times 3 \times -\frac{\sqrt{3}}{2}$

$= \underline{\underline{\frac{-15\sqrt{3}}{2}}}$

(d)

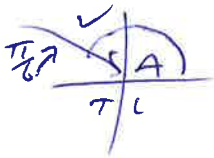


vectors must point away.

$\underline{a} \cdot \underline{b}$

$= |\underline{a}||\underline{b}|\cos\theta$

$= \sqrt{5} \times 2 \times \cos \frac{5\pi}{6}$

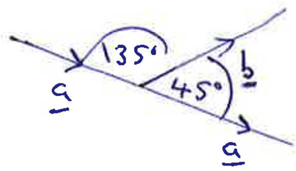


$= \sqrt{5} \times 2 \times -\cos \frac{\pi}{6}$

$= \sqrt{5} \times 2 \times -\frac{\sqrt{3}}{2}$

$= \underline{\underline{-\sqrt{15}}}$

(e)



vectors must point away.

$\underline{a} \cdot \underline{b}$

$= |\underline{a}||\underline{b}|\cos\theta$

$= 4 \times \sqrt{2} \cos 45$

$= 4 \times \sqrt{2} \times \frac{1}{\sqrt{2}}$

$= \underline{\underline{4}}$

(f) $\underline{a} \cdot \underline{b}$

$= |\underline{a}||\underline{b}|\cos\theta$

$= 2 \times 3 \times \cos 120$

$= 2 \times 3 \times -\cos 60$

$= 2 \times 3 \times -\frac{1}{2}$

$= \underline{\underline{-3}}$

6B

② (a) $\underline{u} \cdot \underline{v}$

$$= \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}$$

$= 3 \times 2 + (-2) \times 5 + 1 \times 0$

$= 6 - 10 + 0$

$= \underline{\underline{-4}}$

(b) $\underline{f} \cdot \underline{q}$

$$= \begin{pmatrix} 4 \\ -6 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix}$$

$= 4 \times (-3) + (-6) \times (-2) + (-4) \times 1$

$= -12 + 12 - 4$

$= \underline{\underline{-4}}$

(c) $\underline{a} \cdot \underline{b}$

$$= \begin{pmatrix} 6 \\ -\sqrt{3} \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ \sqrt{2} \\ -4 \end{pmatrix}$$

$= 6 \times 2 + (-\sqrt{3}) \times \sqrt{2} + 1 \times (-4)$

$= 12 - \sqrt{6} - 4$

$= \underline{\underline{8 - \sqrt{6}}}$

(d) $\underline{u} \cdot \underline{v}$

$$= \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$$

$= 2 \times 2 + (-4) \times (-3) + 1 \times 6$

$= 4 + 12 + 6$

$= \underline{\underline{22}}$

(e) $\underline{v} \cdot \underline{w}$

$$= \begin{pmatrix} 0 \\ 2 \\ -\sqrt{5} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ 2\sqrt{3} \end{pmatrix}$$

$= 0 \times 1 + 2 \times 3 + (-\sqrt{5}) \times 2\sqrt{3}$

$= 0 + 6 - 2\sqrt{15}$

$= \underline{\underline{6 - 2\sqrt{15}}}$

(f) $\underline{u} \cdot \underline{v}$

$$= \begin{pmatrix} 2\sqrt{3} \\ 4 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} \sqrt{6} \\ -2 \\ -3 \end{pmatrix}$$

$= 2\sqrt{3} \times \sqrt{6} + 4 \times (-2) + (-3) \times (-3)$

$= 2\sqrt{18} - 8 + 9$

$= 2\sqrt{9}\sqrt{2} + 1$

$= 2 \times 3\sqrt{2} + 1$

$= \underline{\underline{6\sqrt{2} + 1}}$

6B

3(a) $\vec{AB}$

$= \underline{b} - \underline{a}$

$= \begin{pmatrix} 3 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}$

$\vec{AC}$

$= \underline{c} - \underline{a}$

$= \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ -7 \\ -1 \end{pmatrix}$

$\vec{AB} \cdot \vec{AC}$

$= \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 7 \\ -7 \\ -1 \end{pmatrix}$

$= 5 \times 7 + 2 \times (-7) + (-3) \times (-1)$

$= 35 - 14 + 3$

$= \underline{\underline{24}}$

6B

(3) (b) $\vec{AB}$

$= \underline{b} - \underline{a}$

$= \begin{pmatrix} 3 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}$

$\vec{BC}$

$= \underline{c} - \underline{b}$

$= \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -9 \\ 2 \end{pmatrix}$

$\vec{AB} \cdot \vec{BC}$

$= \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -9 \\ 2 \end{pmatrix}$

$= 5 \times 2 + 2 \times (-9) + (-3) \times 2$

$= 10 - 18 - 6$

$= \underline{\underline{-14}}$

6B

4(a) $\overrightarrow{QR}$

$= \underset{\sim}{r} - \underset{\sim}{q}$

$= \begin{pmatrix} -3 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ -7 \end{pmatrix}$

$= \begin{pmatrix} -7 \\ 3 \\ 10 \end{pmatrix}$

$\overrightarrow{PQ}$

$= \underset{\sim}{q} - \underset{\sim}{p}$

$= \begin{pmatrix} 4 \\ 2 \\ -7 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 2 \\ -8 \end{pmatrix}$

$\overrightarrow{QR} \cdot \overrightarrow{PQ}$

$= \begin{pmatrix} -7 \\ 3 \\ 10 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -8 \end{pmatrix}$

$= (-7) \times 1 + 3 \times 2 + 10 \times (-8)$

$= -7 + 6 - 80$

$= \underline{\underline{-81}}$

6B

4(b) $\overrightarrow{RP}$

$= \underset{\sim}{p} - \underset{\sim}{r}$

$= \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -5 \\ -2 \end{pmatrix}$

$\overrightarrow{QP}$

$= \underset{\sim}{p} - \underset{\sim}{q}$

$= \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ -7 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ -2 \\ 8 \end{pmatrix}$

$\overrightarrow{RP} \cdot \overrightarrow{QP}$

$= \begin{pmatrix} 6 \\ -5 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -2 \\ 8 \end{pmatrix}$

$= 6 \times (-1) + (-5) \times (-2) + (-2) \times 8$

$= -6 + 10 - 16$

$= \underline{\underline{-12}}$

6B

(5) $\underline{u} \cdot \underline{v}$

$$= \begin{pmatrix} 2 \\ -1 \\ p \end{pmatrix} \cdot \begin{pmatrix} 2p \\ 3 \\ 1 \end{pmatrix}$$

$$= 2 \times 2p + (-1) \times 3 + p \times 1$$

$$= 4p - 3 + p$$

$$= 5p - 3$$

$$= 22$$

$$5p - 3 = 22$$

$$5p = 25$$

$$\underline{\underline{p = 5}}$$

(6) $\underline{a} \cdot \underline{b}$

$$= \begin{pmatrix} t \\ 5t \\ -8 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$$

$$= 3t + 5t \times 2 + (-8) \times (-4)$$

$$= 3t + 10t + 32$$

$$= 6$$

$$13t + 32 = 6$$

$$13t = -26$$

$$\underline{\underline{t = -2}}$$

6B

7(a) $\underline{a} \cdot (\underline{b} + \underline{c})$

$= \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} \cdot \left(\begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix} \right)$ calculate $\underline{b} + \underline{c}$ first

$= \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -3 \\ -4 \end{pmatrix}$

$= 1 \times 6 + (-3) \times (-3) + 4 \times (-4)$

$= 6 + 9 - 16$

$= \underline{\underline{-1}}$

(b) $\underline{b} \cdot (\underline{a} - \underline{c})$

$= \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} \cdot \left(\begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix} \right)$ calculate $\underline{b} - \underline{c}$ first

$= \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix}$

$= 2 \times (-3) + 0 \times 0 + (-5) \times 3$

$= -6 + 0 - 15$

$= \underline{\underline{-21}}$

6B

7(c)

$$\underline{a} \cdot (\underline{a} + \underline{b} + \underline{c})$$

$$= \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} \cdot \left(\begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix} \right)$$

$$= \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 7 \\ -6 \\ 0 \end{pmatrix}$$

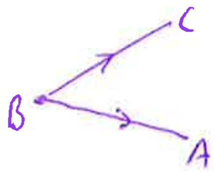
$$= 1 \times 7 + 3 \times (-6) + (-4) \times 0$$

$$= 7 - 18 + 0$$

$$= \underline{\underline{-11}}$$

6C

2(a)



$\vec{BA}$

$= \underset{\sim}{a} - \underset{\sim}{b}$

$= \begin{pmatrix}-2\\3\\5\end{pmatrix} - \begin{pmatrix}4\\0\\7\end{pmatrix}$

$= \begin{pmatrix}-6\\3\\-2\end{pmatrix}$

$\vec{BC}$

$= \underset{\sim}{c} - \underset{\sim}{b}$

$= \begin{pmatrix}2\\2\\1\end{pmatrix} - \begin{pmatrix}4\\0\\7\end{pmatrix}$

$= \begin{pmatrix}-2\\2\\-6\end{pmatrix}$

$\vec{BA} \cdot \vec{BC}$

$= (-6) \times (-2) + 3 \times 2 + (-2) \times (-6)$

$= 12 + 6 + 12$

$= \underline{\underline{30}}$

$|\vec{BA}| = \sqrt{(-6)^2 + 3^2 + (-2)^2}$

$= 7$

$|\vec{BC}| = \sqrt{(-2)^2 + 2^2 + (-6)^2}$

$= \sqrt{44}$

$\vec{BA} \cdot \vec{BC} = |\vec{BA}| |\vec{BC}| \cos\theta$

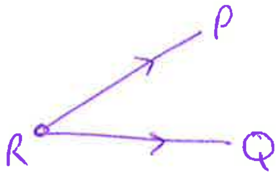
$\cos\theta = \dfrac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}| |\vec{BC}|}$

$\cos\theta = \left(\dfrac{30}{7\sqrt{44}}\right)$

$\underline{\underline{\theta = 49.8^\circ}}$ (0.868 rad)

6C

(3)



$\vec{RP}$

$= \underset{\sim}{p} - \underset{\sim}{r}$

$= \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ -9 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 2 \\ 9 \end{pmatrix}$

$\vec{RQ}$

$= \underset{\sim}{q} - \underset{\sim}{r}$

$= \begin{pmatrix} 5 \\ -6 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ -9 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -6 \\ 10 \end{pmatrix}$

$\vec{RP} \cdot \vec{RQ}$

$= (-1) \times 1 + 2 \times (-6) + 9 \times 10$

$= -1 - 12 + 90$

$= \underline{\underline{77}}$

$|\vec{RP}|$

$= \sqrt{(-1)^2 + (2)^2 + 9^2}$

$= \sqrt{86}$

$|\vec{RQ}|$

$= \sqrt{1^2 + (-6)^2 + 10^2}$

$= \sqrt{137}$

$\vec{RP} \cdot \vec{RQ} = |\vec{RP}||\vec{RQ}| \cos\theta$

$\cos\theta = \dfrac{\vec{RP} \cdot \vec{RQ}}{|\vec{RP}||\vec{RQ}|}$

$\cos\theta = \left(\dfrac{77}{\sqrt{86}\sqrt{137}} \right)$

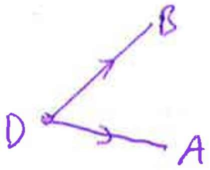
$\theta = \cos^{-1}\left(\dfrac{77}{\sqrt{86}\sqrt{137}} \right)$

$\underline{\underline{\theta = 44.8^\circ}}$ (0.782 rad)

6C

(5) (a) $B(8,8,0)$

(b)



$\vec{DA}$

$= \underline{a} - \underline{d}$

$= \begin{pmatrix} 8 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 9 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -4 \\ -9 \end{pmatrix}$

$\vec{DB}$

$= \underline{b} - \underline{d}$

$= \begin{pmatrix} 8 \\ 8 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 9 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 4 \\ -9 \end{pmatrix}$

$\vec{DA} \cdot \vec{DB}$

$= 4 \times 4 + (-4) \times 4 + (-9) \times (-9)$

$= 16 - 16 + 81$

$= \underline{\underline{81}}$

$|\vec{DA}| = \sqrt{(4)^2 + (+4)^2 + (-9)^2}$

$= \sqrt{113}$

$\vec{DB} = \sqrt{(4)^2 + (-4)^2 + (-9)^2}$

$= \sqrt{113}$

$\vec{DA} \cdot \vec{DB} = |\vec{DA}||\vec{DB}| \cos\theta$

$\cos\theta = \dfrac{\vec{DA} \cdot \vec{DB}}{|\vec{DA}|\,|\vec{DB}|}$

$\cos\theta = \dfrac{81}{\sqrt{113}\sqrt{113}}$

$\theta = \cos^{-1}\left(\dfrac{81}{113}\right)$

$\underline{\underline{\theta = 44.2^\circ}}$ (0.772 rads)

6C

11 (a)

$\vec{AJ}$

$= \underline{j} - \underline{a}$

$= \begin{pmatrix} 3 \\ -5 \\ h \end{pmatrix} - \begin{pmatrix} 9 \\ 4 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -6 \\ -9 \\ h \end{pmatrix}$

$|\vec{AJ}| = \sqrt{181}$

$\sqrt{(-6)^2 + (-9)^2 + h^2} = \sqrt{181}$

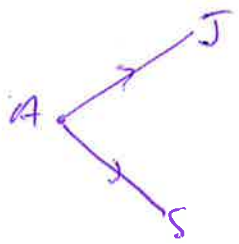
$\sqrt{117 + h^2} = \sqrt{181}$

$h^2 = 64$

$\underline{\underline{h = 8}}$

(b) $\vec{AJ}$

$= \begin{pmatrix} -6 \\ -9 \\ 8 \end{pmatrix}$



$\vec{AS}$

$= \underline{s} - \underline{a}$

$= \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 4 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -7 \\ 1 \\ -3 \end{pmatrix}$

$\vec{AJ} \cdot \vec{AS} = |\vec{AJ}||\vec{AS}| \cos\theta$

$|\vec{AJ}| = \sqrt{(-6)^2 + (-9)^2 + 8^2}$

$= \sqrt{181}$

$|\vec{AS}| = \sqrt{59}$

$\vec{AJ} \cdot \vec{AS} = (-6)\times(-7) + (-9)\times 1 + 8\times(-3$

$= 42 - 9 - 24$

$= 9$

$\cos\theta = \dfrac{\vec{AJ} \cdot \vec{AS}}{|\vec{AJ}|\,|\vec{AS}|}$

$= \dfrac{9}{\sqrt{181}\sqrt{59}}$

$\theta = \cos^{-1}\left(\dfrac{9}{\sqrt{181}\sqrt{59}}\right)$

$\underline{\underline{\theta = 85.0^\circ}}$ (1.484 rads)

6C

12 (a).

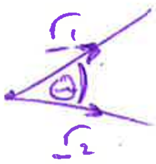
$$\underline{r_1} = \begin{pmatrix} 5 \\ 12 \\ 9 \end{pmatrix} + \begin{pmatrix} -3 \\ 4 \\ 16 \end{pmatrix} + \begin{pmatrix} 4 \\ -5 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 11 \\ 33 \end{pmatrix}$$

(b)

$$\underline{r_2} = \begin{pmatrix} 5 \\ 12 \\ 9 \end{pmatrix} + \begin{pmatrix} -3 \\ 4 \\ 16 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 16 \\ 25 \end{pmatrix}$$



$$\underline{r_1} \cdot \underline{r_2}$$

$$= 6 \times 2 + 11 \times 16 + 33 \times 25$$

$$= 12 + 176 + 825$$

$$= 1013$$

$$|\underline{r_1}| = \sqrt{6^2 + 11^2 + 33^2}$$

$$= \sqrt{1246}$$

$$|\underline{r_2}| = \sqrt{2^2 + 16^2 + 25^2}$$

$$= \sqrt{885}$$

$$\underline{r_1} \cdot \underline{r_2} = |\underline{r_1}||\underline{r_2}| \cos\theta$$

$$\cos\theta = \frac{\underline{r_1} \cdot \underline{r_2}}{|\underline{r_1}|\,|\underline{r_2}|}$$

$$\theta = \cos^{-1}\left(\frac{1013}{\sqrt{885}\sqrt{1246}}\right)$$

$$\underline{\underline{\theta = 15.3^\circ}} \ (0.267 \text{ rads})$$

6D

1(a)

$\underline{u} \cdot \underline{v}$

$= 3 \times 5 + 2 \times (-3) + (-1) \times 9$

$= 15 - 6 - 9$

$= 0$

$\underline{u} \cdot \underline{v} = 0 \quad \therefore \underline{u}$ and $\underline{v}$ are perpendicular

(b) $\underline{c} \cdot \underline{d}$

$= 6 \times 5 + 8 \times 3 + 9 \times -6$

$= 30 + 24 - 54$

$= 0$

$\underline{c} \cdot \underline{d} = 0 \quad \therefore \underline{c}$ and $\underline{d}$ are perpendicular

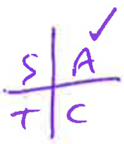
2 (a) $\underline{a} \cdot \underline{b}$

$= 3 \times 2 + 2 \times 4 + (-4) \times 1$

$= 6 + 8 - 4$

$= 10$

as $\underline{a} \cdot \underline{b} > 0$ the angle between $\underline{a}$ and $\underline{b}$ is acute



$\cos\theta = \dfrac{\underline{a} \cdot \underline{b}}{|\underline{a}|\,|\underline{b}|}$

$\underline{a} \cdot \underline{b} > 0$

$|\underline{a}| > 0 \qquad \therefore \cos\theta > 0$

$|\underline{b}| > 0 \qquad 0 < \theta < 90^\circ$

6D

2(c) $\underline{u} \cdot \underline{v}$

$= (-2)\times 4 + 5 \times 1 + (-2) \times 6$

$= -8 + 5 - 12$

$= -15$

as $\underline{u} \cdot \underline{v} < 0$ the angle between $\underline{u}$ and $\underline{v}$ will be obtuse ~~acute~~.

$\dfrac{\underline{u} \cdot \underline{v}}{|\underline{u}|\,|\underline{v}|} < 0 \quad \therefore \quad \cos\theta < 0$

S ✓	A
T ✓	C

we will be looking for an angle in the 2^{nd} quadrant as $\cos\theta$ is negative, $\therefore \theta$ is obtuse

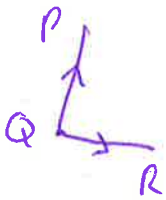
note: angle in the 3^{rd} quadrant from

S ✓	A
T ✓	C

would be the reflex angle

6D

(3)



$\overrightarrow{QP}$

$= \underset{\sim}{p} - \underset{\sim}{q}$

$= \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 8 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ -5 \\ -3 \end{pmatrix}$

$\overrightarrow{QR}$

$= \underset{\sim}{r} - \underset{\sim}{q}$

$= \begin{pmatrix} 3 \\ -1 \\ 13 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 8 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}$

$\overrightarrow{QP} \cdot \overrightarrow{QR}$

$= 0 \times 1 + (-5) \times (-3) + (-3) \times (5)$

$= 0 + 15 - 15$

$= \underline{\underline{0}}$

$\overrightarrow{QP} \cdot \overrightarrow{QR} = 0 \therefore \overrightarrow{QP}$ is perpendicular to $\overrightarrow{QR}$ which means that $\triangle PQR$ will be right angled at Q.

6D

(6) $\underline{a} \cdot \underline{b} = 0$ as $\underline{a}$ is perpendicular to $\underline{b}$

$p \times 2 + 3 \times (-1) + 2 \times 2p = 0$

$6p - 3 = 0$

$6p = 3$

$p = \frac{1}{2}$

(7) $\underline{p} \cdot \underline{q} = 0$ as $\underline{p} \perp \underline{q}$

$2x \times x + -7 \times x + 3 \times -5 = 0$

$2x^2 - 7x - 15 = 0$

$(2x + 3)(x - 5) = 0$

$2x + 3 = 0$ $\quad x - 5 = 0$

$2x = -3$ $\quad x = 5$

$x = \frac{-3}{2}$

(8) $\vec{AB}$

$= \underline{b} - \underline{a}$

$= \begin{pmatrix} -1 \\ 4 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$

6D)

8 continued

$\vec{CD}$

$= \underline{d} - \underline{c}$

$= \begin{pmatrix} -1 \\ 2 \\ 8 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$

consider $\vec{AB}$

$\begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ p \\ q \end{pmatrix} = 0$ as $\vec{AB} \perp \begin{pmatrix} 1 \\ p \\ q \end{pmatrix}$

$2 + 3p + 6q = 0$ ①

consider $\vec{CD}$

$\begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ p \\ q \end{pmatrix} = 0$ as $\vec{CD} \perp \begin{pmatrix} 1 \\ p \\ q \end{pmatrix}$

$1 + 2p + 5q = 0$ ②

$2 + 3p + 6q = 0$ ①

$4 + 6p + 12q = 0$ ③ $(2 \times ①)$

$3 + 6p + 15q = 0$ ④ $(3 \times ②)$

$-1 \quad + 3q = 0$ ④ $-$ ③

$3q = 1$

$\underline{\underline{q = \frac{1}{3}}}$

sub $q = \frac{1}{3}$ into ①

$1 + 2p + 5\left(\frac{1}{3}\right) = 0$

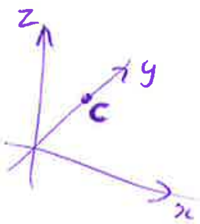
$2p = -\frac{5}{3} - 1$

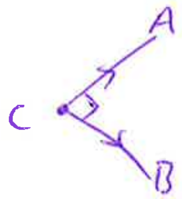
$2p = -\frac{8}{3}$

$\underline{\underline{p = -\frac{4}{3}}}$

6D

(10) $C(0, y, 0)$





$\overrightarrow{CA}$

$= \underline{a} - \underline{c}$

$= \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ -y \\ 1 \end{pmatrix}$

$\overrightarrow{CB}$

$= \underline{b} - \underline{c}$

$= \begin{pmatrix} 5 \\ 2 \\ 7 \end{pmatrix} - \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 2-y \\ 7 \end{pmatrix}$

$\overrightarrow{CA} \cdot \overrightarrow{CB} = 0$ as $\overrightarrow{CA} \perp \overrightarrow{CB}$

$\begin{pmatrix} -3 \\ -y \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 2-y \\ 7 \end{pmatrix} = 0$

$-3 \times 5 + (-y)(2-y) + 1 \times 7 = 0$

$-15 + -2y + y^2 + 7 = 0$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$y - 4 = 0$ $\quad$ $y + 2 = 0$

$\underline{\underline{y = 4}}$ $\quad$ $\underline{\underline{y = -2}}$

6E

(2) (a) $\underline{a} \cdot (\underline{a} + \underline{b})$

$= \underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{b}$

$= a^2 + |\underline{a}| |\underline{b}| \cos\theta$

$= 5^2 + (5)(3) \cos 60$

$= 25 + (15)(\frac{1}{2})$

$= 25 + \frac{15}{2}$

$= \frac{65}{2}$

(c) $(\underline{a} + 2\underline{b}) \cdot (\underline{a} - 3\underline{b})$

$= \underline{a} \cdot \underline{a} - \underline{a} \cdot 3\underline{b} + 2\underline{b} \cdot \underline{a} - 6 \underline{b} \cdot \underline{b}$

$= a^2 - 3|\underline{a}||\underline{b}| \cos\theta + 2|\underline{a}||\underline{b}| \cos\theta - 6b^2$

$= a^2 - |\underline{a}| |\underline{b}| \cos\theta + 6b^2$

$= (5)^2 - (5)(3) \cos 60 - 6(3)^2$

$= 25 - \frac{15}{2} - 54$

$= \frac{-73}{2}$

GE

(4) $p \cdot (p + q) = 9$

$p \cdot p + p \cdot q = 9$

$p^2 + |p||q| \cos\theta = 9$

$2^2 + 2 \times 5 \times \cos\theta = 9$

$4 + 10\cos\theta = 9$

$10\cos\theta = 5$

$\cos\theta = \frac{1}{2}$

$\theta = \cos^{-1}(\frac{1}{2})$

$\theta = 60°$

(5) $\underline{u} \cdot (\underline{u} + \underline{v} + \underline{w})$

$= \underline{u} \cdot \underline{u} + \underline{u} \cdot \underline{v} + \underline{u} \cdot \underline{w}$

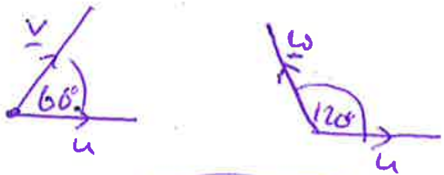
$= u^2 + |\underline{u}||\underline{v}| \cos\theta + |\underline{u}||\underline{w}| \cos\theta$

$= u^2 + |\underline{u}||\underline{v}| \cos 60 + |\underline{u}||\underline{w}| \cos 120$

$= u^2 + |\underline{u}||\underline{v}| \cos 60 + |\underline{u}||\underline{w}|(-\cos 60)$

$= u^2$

as equilateral triangle $|\underline{u}| = |\underline{v}| = |\underline{w}|$



Vectors must point away from corner

$\cos 120 = -\cos 60$

6E

(8) $(\underline{u}+\underline{v})\cdot|\underline{u}+\underline{v}|$

$= \underline{u}\cdot\underline{u} + \underline{u}\cdot\underline{v} + \underline{v}\cdot\underline{u} + \underline{v}\cdot\underline{v}$

$= \underline{u}\cdot\underline{u} + 2\underline{u}\cdot\underline{v} + \underline{v}\cdot\underline{v}$

$\Rightarrow 2\underline{u}\cdot\underline{v} = 0$

$\underline{u}\cdot\underline{v} = 0$

As $\underline{u}\cdot\underline{v} = 0$ $\underline{u}$ is perpendicular to $\underline{v}$ $\therefore \Delta ABC$ is right angled at B.

(10) $\underline{a}\cdot(\underline{b}+\underline{c})$

$= \underline{a}\cdot\underline{b} + \underline{a}\cdot\underline{c}$

$= |\underline{a}||\underline{b}|\cos\theta + |\underline{a}||\underline{c}|\cos\theta$

$= 5\times 4\times\cos 30 + \cancelto{0}{|\underline{a}||\underline{c}|\cos 90}$

$\underline{a} \perp \underline{c}$ so $|\underline{a}||\underline{c}|\cos 90 = 0$

$= 20\times\frac{\sqrt{3}}{2}$

$= \underline{\underline{10\sqrt{3}}}$

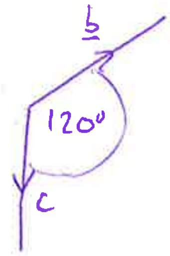
$\underline{c}\cdot(\underline{a}-\underline{b})$

$= \cancelto{0}{\underline{c}\cdot\underline{a}} - \underline{c}\cdot\underline{b}$

$\underline{c} \perp \underline{a}$ $\therefore$ $\underline{c}\cdot\underline{a} = 0$

$= -\underline{c}\cdot\underline{b}$

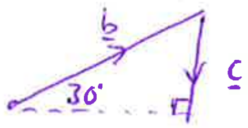
$= -|\underline{c}||\underline{b}|\cos 120$



• vectors must point away

6E

(10) continued



$$\sin 30 = \frac{|\underset{\sim}{c}|}{|\underset{\sim}{b}|}$$

$$|\underset{\sim}{b}| \sin 30 = |\underset{\sim}{c}|$$

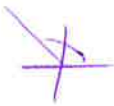
$$4 \sin 30 = |\underset{\sim}{c}|$$

$$4 \times \tfrac{1}{2} = |\underset{\sim}{c}|$$

$$|\underset{\sim}{c}| = 2$$

$$- |\underset{\sim}{c}||\underset{\sim}{b}| \cos 120$$

$$= -(2)(4) \cos 120$$



$$\cos 120 = -\cos 60$$

$$= -(2)(4)(-\cos 60)$$

$$= -(2)(4)\left(-\tfrac{1}{2}\right)$$

$$= \underline{\underline{4}}$$