

Partial Fractions - Solutions.

$$1. \quad \frac{4x+9}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$= \frac{A(x-3) + B(x-2)}{(x-2)(x-3)}$$

$$4x+9 = A(x-3) + B(x-2)$$

$$\text{Let } x=3 \quad 21 = B$$

$$\text{Let } x=2 \quad 17 = -A$$

$$\frac{4x+9}{(x-2)(x-3)} = \frac{21}{x-3} - \frac{17}{x-2}$$

$$2. \quad \frac{1}{x(x-3)(x+3)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+3}$$

$$= \frac{A(x-3)(x+3) + Bx(x+3) + Cx(x-3)}{x(x-3)(x+3)}$$

$$1 = A(x-3)(x+3) + Bx(x+3) + Cx(x-3)$$

$$\text{Let } x=3 \quad 1 = 18B$$

$$B = \frac{1}{18}$$

$$\text{Let } x=-3 \quad 1 = 18C$$

$$C = \frac{1}{18}$$

$$\text{Let } x=0 \quad 1 = -9A$$

$$A = -\frac{1}{9}$$

$$\frac{1}{x(x-3)(x+3)} = \frac{1}{18(x-3)} + \frac{1}{18(x+3)} - \frac{1}{9x}$$

$$3. \frac{8x-1}{(x-2)(x^2+1)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+1}$$

$$= \frac{A(x^2+1) + (Bx+C)(x-2)}{(x-2)(x^2+1)}$$

$$8x-1 = A(x^2+1) + (Bx+C)(x-2)$$

$$\text{Let } x=2 \quad 15 = 5A$$

$$A = 3$$

$$\text{Let } x=0 \quad -1 = A - 2C$$

$$-4 = -2C$$

$$C = 2$$

$$\text{Let } x=1 \quad 7 = 2A + (B+C)(-1)$$

$$7 = 6 - B - 2$$

$$3 = -B$$

$$B = -3$$

$$\frac{8x-1}{(x-2)(x^2+1)} = \frac{3}{x-2} + \frac{2-3x}{x^2+1}$$

$$4. \frac{5x^2-2x-1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$= \frac{A(x^2+1) + (Bx+C)(x+1)}{(x+1)(x^2+1)}$$

$$5x^2 - 2x - 1 = A(x^2+1) + (Bx+C)(x+1)$$

$$\text{Let } x=-1 \quad 6 = 2A$$

$$A = 3$$

$$\text{Let } x=0 \quad -1 = A + C$$

$$-4 = C$$

$$\text{Let } x=1 \quad 2 = 2A + (B+C)(2)$$

$$2 = 6 + 2B - 8$$

$$4 = 2B$$

$$B = 2$$

$$\frac{5x^2-2x-1}{(x+1)(x^2+1)} = \frac{3}{x+1} + \frac{2x-4}{x^2+1}$$

$$5. \frac{x+4}{(x+1)(x-2)^2} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$= \frac{A(x-2)^2 + B(x+1)(x-2) + C(x+1)}{(x+1)(x-2)^2}$$

$$x+4 = A(x-2)^2 + B(x+1)(x-2) + C(x+1)$$

$$\text{Let } x=2 \quad 6 = 3C$$

$$C = 2$$

$$\text{Let } x=-1 \quad 3 = 9A$$

$$A = \frac{1}{3}$$

$$\text{Let } x=0 \quad 4 = 4A - 2B + C$$

$$4 = \frac{4}{3} - 2B + 2$$

$$\frac{2}{3} = -2B \Rightarrow B = -\frac{1}{3}$$

$$\frac{x+4}{(x+1)(x-2)^2} = \frac{1}{3(x+1)} - \frac{1}{3(x-2)} + \frac{2}{(x-2)^2}$$

$$6. \frac{x-3-2x^2}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

$$= \frac{Ax(x-1) + B(x-1) + Cx^2}{x^2(x-1)}$$

$$x-3-2x^2 = Ax(x-1) + B(x-1) + Cx^2$$

$$\text{Let } x=0 \quad -3 = -B$$

$$B = 3$$

$$\text{Let } x=1 \quad -4 = C$$

$$\text{Let } x=-1 \quad -6 = 2A - 2B + C$$

$$-6 = 2A - 6 - 4$$

$$4 = 2A$$

$$A = 2$$

$$\frac{x-3-2x^2}{x^2(x-1)} = \frac{2}{x} + \frac{3}{x^2} - \frac{4}{x-1}$$

7.

$$x^2 + x - 12 \overline{) \begin{array}{r} x^2 + 3x \\ -(x^2 + x - 12) \\ \hline 2x + 12 \end{array}}$$

$$\frac{x(x+3)}{x^2+x-12} = 1 + \frac{2x+12}{x^2+x-12}$$

$$= 1 + \frac{A}{(x+4)} + \frac{B}{(x-3)}$$

$$= 1 + \frac{A(x-3) + B(x+4)}{(x+4)(x-3)}$$

$$2x + 12 = A(x-3) + B(x+4)$$

$$\text{Let } x=3 \quad 18 = 7B \\ B = \frac{18}{7}$$

$$\text{Let } x=-4 \quad 4 = -7A \\ A = \frac{-4}{7}$$

$$\frac{x(x+3)}{x^2+x-12} = 1 + \frac{18}{7(x-3)} - \frac{4}{7(x+4)}$$

8.

$$x^2 + 3x - 4 \overline{) \begin{array}{r} x^3 \\ -(x^3 + 3x^2 - 4x) \\ \hline -3x^2 + 4x \\ -(-3x^2 - 9x + 12) \\ \hline 13x - 12 \end{array}}$$

$$\frac{x^3}{(x+4)(x-1)} = x-3 + \frac{13x-12}{(x+4)(x-1)}$$

$$\frac{13x - 12}{(x+4)(x-1)} = \frac{A}{x+4} + \frac{B}{x-1}$$
$$= \frac{A(x-1) + B(x+4)}{(x+4)(x-1)}$$

$$13x - 12 = A(x-1) + B(x+4)$$

$$\text{Let } x=1 \quad 1 = 5B$$
$$B = \frac{1}{5}$$

$$\text{Let } x=-4 \quad -64 = -5A$$

$$A = \frac{64}{5}$$

$$\frac{x^3}{(x+4)(x-1)} = x - 3 + \frac{64}{5(x+4)} + \frac{1}{5(x-1)}$$