

Homework 9 solutions

1

$$1) \frac{2x^2 - 3x + 2}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} \quad \checkmark$$

$$\frac{2x^2 - 3x + 2}{x^2(x-1)} = \frac{Ax(x-1)}{x^2(x-1)} + \frac{B(x-1)}{x^2(x-1)} + \frac{Cx^2}{x^2(x-1)}$$

$$\Rightarrow 2x^2 - 3x + 2 = Ax(x-1) + B(x-1) + Cx^2 \quad \checkmark$$

$$x=1 \Rightarrow 1 = C \quad \underline{\underline{C=1}} \quad \checkmark$$

$$x=0 \Rightarrow 2 = -B \quad \underline{\underline{B=-2}} \quad \checkmark$$

$$2x^2 - 3x + 2 = Ax(x-1) - 2(x-1) + x^2$$

$$x=2 \Rightarrow \begin{aligned} 4 &= 2A - 2 + 4 \\ 2A &= 2 \\ \underline{\underline{A=1}} & \quad \checkmark \end{aligned} \quad \frac{1}{x} - \frac{2}{x^2} + \frac{1}{x-1} = \frac{2x^2 - 3x + 2}{x^2(x-1)}$$

$$\text{or } \frac{2x^2 - 3x + 2}{x^2(x-1)} = \frac{Ax+B}{x^2} + \frac{C}{x-1} \quad \checkmark$$

$$\frac{2x^2 - 3x + 2}{x^2(x-1)} = \frac{(Ax+B)(x-1)}{x^2(x-1)} + \frac{Cx^2}{x^2(x-1)} \quad \checkmark$$

$$x=0 \quad 2 = -B \Rightarrow \underline{\underline{B=-2}} \quad \checkmark$$

$$x=1 \quad 1 = C \Rightarrow \underline{\underline{C=1}} \quad \checkmark$$

$$2x^2 - 3x + 2 = (Ax-2)(x-1) + x^2$$

$$x=2 \quad \begin{aligned} 4 &= 2A - 2 + 4 \\ \underline{\underline{A=1}} & \quad \checkmark \end{aligned}$$

2) $y = \ln(1 + \sin x)$

$\frac{dy}{dx} = \frac{1}{1 + \sin x} \times \cos x$

$\frac{dy}{dx} = \frac{\cos x}{1 + \sin x}$

$\frac{d^2y}{dx^2} = \frac{-\sin x (1 + \sin x) - \cos x \cdot \cos x}{(1 + \sin x)^2}$

$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1 + \sin x)^2}$

$= \frac{-(\sin x + \sin^2 x + \cos^2 x)}{(1 + \sin x)^2}$

$[\sin^2 x + \cos^2 x = 1]$

$= \frac{-\cancel{(1 + \sin x)}}{(1 + \sin x)^{\cancel{2}}}$

$\frac{d^2y}{dx^2} = \frac{-1}{1 + \sin x}$

3)

$$x^2 + 4xy + y^2 + 11 = 0$$

$$2x + 4y + 4x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$x = -2 \quad y = 3 \Rightarrow -4 + 12 - 8 \frac{dy}{dx} + 6 \frac{dy}{dx} = 0$$

$$2 \frac{dy}{dx} = 8$$

$$\frac{dy}{dx} = 4$$

$$2x + 4y + 4x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$2 + 4 \frac{dy}{dx} + 4 \frac{dy}{dx} + 4x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} \left(\frac{dy}{dx} \right) + 2y \frac{d^2y}{dx^2} = 0$$

$$x = -2 \quad y = 3$$

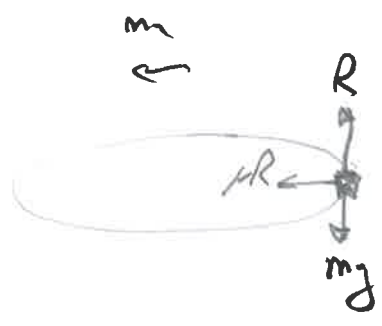
$$\frac{dy}{dx} = 4$$

$$2 + 16 + 16 - 8 \frac{d^2y}{dx^2} + 32 + 6 \frac{d^2y}{dx^2} = 0$$

$$2 \frac{d^2y}{dx^2} = 66$$

$$\frac{d^2y}{dx^2} = 33$$

4) $\omega = \frac{v}{r}$
 $\omega = \frac{2}{3} \text{ rads}^{-1}$ ✓



$ma = \mu R$ $R = mg$ ✓
 $\mu a = \mu mg$ ✓
 $\omega^2 r = \mu g$
 $\mu = \frac{\omega^2 r}{g}$ $r = 4m$
 $\mu = 0.181$ ✓



$R = ma$ $\mu R = mg$ ✓
 $\mu ma = mg$ ✓
 $\frac{\mu v^2}{r} = g$
 $\mu = \frac{gr}{v^2}$
 $\mu = 0.109$ ✓