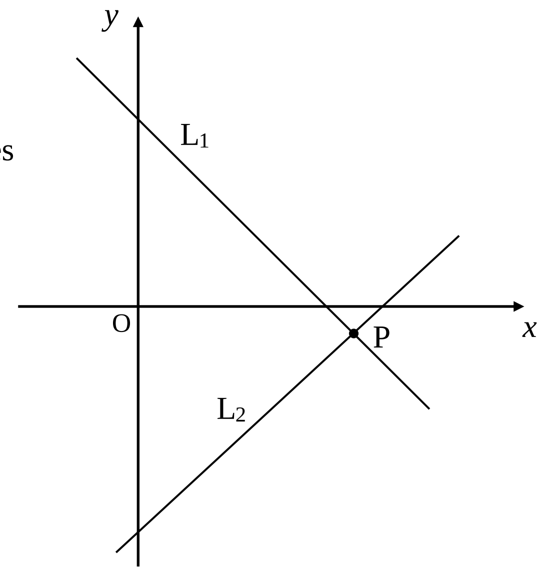
TEST D

1. (a) The line L_1 passes through the point (5, 2) and makes an angle of 135^o with the positive direction of the x-axis.

Find the equation of L_1. **(3)**

(b) The line L_2 is perpendicular to L_1 and passes through the point (6, -3).

Find the coordinates of P, the point of intersection of L_1 and L_2. **(4)**



2. Mr Smith, a Biology teacher, had a feeling that his locust colony was dying off.
One day he counted the locusts and discovered that there were only 3600 left and that he was losing them at a rate of 15% each week! He decided that something had to be done. He bought a fresh supply of live locusts and started adding them to the colony at the rate of 240 each week. The new locusts were introduced at the **end** of each week.

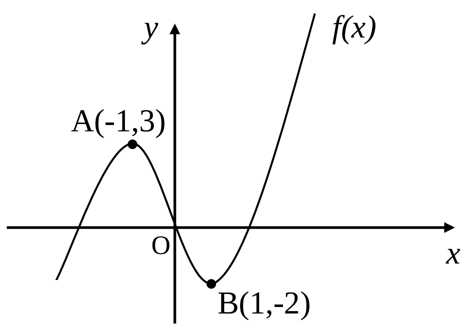
(a) Set up a recurrence relation to illustrate Mr Smith's locust situation. **(2)**

Mr Smith decided that if the number of locusts in the colony fell below 1800 he would need to abandon the colony and start again.

(b) **Over the long term**, would Mr Smith ever have to abandon his colony?
You must show all relevant working and give a reason for your answer. **(3)**

3. Part of the graph of $y = f(x)$ is shown in the diagram.

Sketch the graph of $y = f'(x) + 2$, marking clearly any known points.



(4)

4. Two functions are defined on suitable domains as $f(x) = 4x + 5$ and $g(x) = x^2 + 3$.

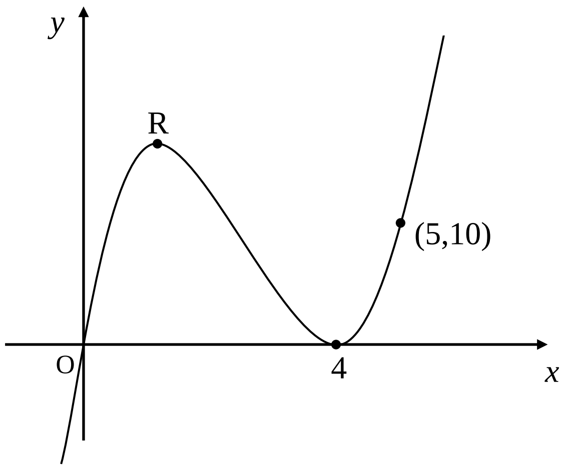
(a) Show clearly that $h(x) = 16x^2 + 40x + 28$, where $h(x) = g(f(x))$. **(2)**

(b) Find the value(s) of x for which $h(x) = 4$. **(3)**

5. (a) The curve in the diagram has its equation in the form $y = k\,x(x-4)^2$, where k is a constant.

The curve passes through the point (5,10).

Find the value of k. **(2)**



(b) Find the equation of the tangent to this curve at the point where $x = 1$. **(5)**

(c) Find the ***x*-coordinate** of R, the maximum turning point of the graph. **(3)**

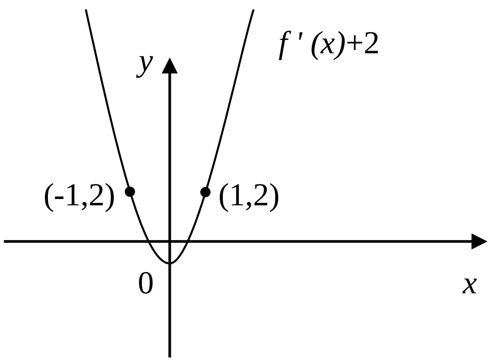
6. Solve algebraically the equation

$$2\sin 2x^{o} + \sqrt{3} = 0 \quad \text{for} \quad 0 < x \leq 360.$$ **(4)**

7. Given that $f(x) = \cos 3x + \sin^2 x$, show that $f'\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$. **(4)**

[END OF QUESTION PAPER]

SET E **Marking Scheme - UNIT 1**

	Give 1 mark for each •	Illustration(s) for awarding each mark
1. (a)	**ans:** $y + x = 7$ **3 marks** •[1] knowing to find tan of angle •[2] substituting into $y - b = x(m - a)$ •[3] re-arranging to acceptable form	 •[1] tan 135° = -1= m •[2] $y - 2 = -1(x - 5)$ •[3] $y + x = 7$ (or equivalent)
(b)	**ans: (8, -1)** **4 marks** •[1] knowing $m_1 \times m_2 = -1$ •[2] substituting & re-arranging •[3] knowing to use sim.eqs. •[4] solving to answer	 •[1] $m_{perp} = 1$ •[2] $y - x = -9$ •[3] evidence •[4] (8, -1)
2. (a)	**Ans:** $U_{n+1} = 0{\cdot}85\ U_n + 240$ **2 marks** •[1] using correct multiplier •[2] adding 240	 •[1] $0{\cdot}85U_n$ •[2] +240
(b)	**Ans: Yes** **3 marks** •[1] stating that limit exists •[2] finding limit •[3] valid conclusion with reason	 •[1] $-1<0{\cdot}85<1$ •[2] 240/0·15 or equivalent………1600 •[3] yes, number of locusts between 1360 and 1600 so less than 1800.
3.	**Ans:** **4 marks**  •[1] knowing to draw $f'(x)$ first. •[2] correct shape of graph. •[3] knowing to move 2 units in y-direction •[4] showing 2 relevant points	*This is only one of a whole family of acceptable curves. Most pupils will probably have min. turning point at origin … but not determinable from given info., however perfectly acceptable.* •[1] evidence of different shaped graph •[2] parabolic shape •[3] moves roots two up •[4] (-1,2) and (1,2) marked

	Give 1 mark for each •	Illustration(s) for awarding each mark
4. (a)	**ans: $16x^2+40x+28$** **2 marks**	
	•¹ knowing correct order of substitution	•¹ $g(f(x))=g(4x+5)=(4x+5)^2+3$
	•² simplifying	•² $16x^2+40x+28$
(b)	**ans: -1 or -1·5** **3 marks**	
	•¹ knowing to equate to 4	•¹ $16x^2+40x+28=4$
	•² re-arranging and factorising	•² $8(2x+3)(x+1)=0$
	•³ solving to answer	•³ $x=-1\cdot5$ or -1
5. (a)	**Ans: proof** **2 marks**	
	•¹ substituting (5,10)	•¹ $10 = k(5)(1)^2$
	•² finding value of '*k*'	•² $k = 2$
(b)	**Ans: y = 6*x* + 12** **5 marks**	
	•¹ preparing to differentiate	•¹ $2x^3-16x^2+32x$
	•² carrying out differentiation	•² $6x^2-32x+32$
	•³ knowing to sub. $x = 1$ to find m	•³ $m = 6$
	•⁴ knowing to find point on line	•⁴ (1, 18)
	•⁵ substituting into $y - b = m(x - a)$ and arranging	•⁵ $y-18=6(x-1)$
(c)	**Ans: $\frac{4}{3}$** **3 marks**	
	•¹ knowing to use derivative = 0	•¹ $\frac{dy}{dx}=0$
	•² factorising	•² $2(3x-4)(x-4)=0$
	•³ solving and discarding x = 4	•³ $x=\frac{4}{3}$ or ~~4~~

	Give 1 mark for each •	Illustration(s) for awarding each mark
6.	**Ans: {120°, 150°, 300°, 330°} 4 marks**	
	•[1] for making $\sin 2x$ the subject	•[1] $\sin 2x = -\frac{\sqrt{3}}{2}$
	•[2] for first two double angles	•[2] $2x = 240^\circ$, 300°.......
	•[3] for second two double angles	•[3] $2x = 600^\circ, 660^\circ$
	•[4] for dividing by 2	•[4] $x = 120^\circ, 150^\circ, 300^\circ, 330^\circ$
7.	**ans: proof 4 marks**	
	•[1] first part of derivative	•[1] $-3\sin 3x$..........
	•[2] second part of derivative	•[2] …….+ $2\sin x\cos x$ [or $\sin 2x$]
	•[3] knowing to substitute $\frac{\pi}{3}$	•[3] $-3\sin\pi + \sin\frac{2\pi}{3}$
	•[4] evaluating to answer	•[4] $-3\times 0 + \frac{\sqrt{3}}{2}$
		Total 39 marks