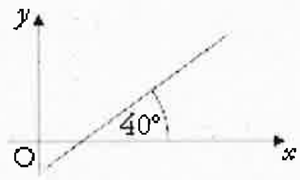
Applications Assessment Standard 1.1

1.(i) A line passes through the points A(4, -3) and B(-6, 2). Find the equation of this line.

(ii) A line makes an angle of 40^0 with the positive direction of the x-axis, as shown in the diagram. Find the equation of this line.



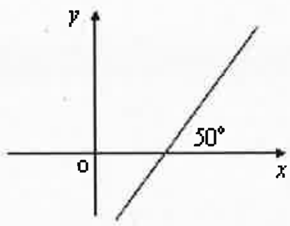
(iii) A line, L, has equation $y = 4x + 1$.

Write down the equation of a line, passing through (2, 1), which is:

(a) Parallel to L (b) Perpendicular to L

2.(i) A line passes through the points A(2, -7) and B(6, 1). Find the equation of this line.

(ii) A line makes an angle of 50^0 with the positive direction of the x-axis, as shown in the diagram. Find the equation of this line.



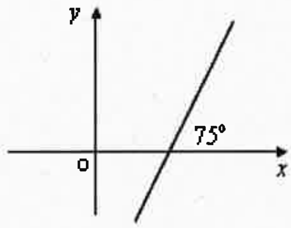
(iii) A line, L, has equation $y = -3x - 1$.

Write down the equation of a line, passing through (4, 2), which is:

(a) Parallel to L (b) Perpendicular to L

3.(i) A line passes through the points A(-1, 3) and B(-4, 2). Find the equation of this line.

(ii) A line makes an angle of 75^0 with the positive direction of the x-axis, as shown in the diagram. Find the equation of this line.



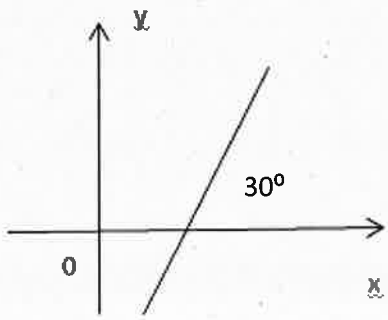
(iii) A line, L, has equation $y = \frac{1}{4}x + 1$.

Write down the equation of a line, passing through (3, -1), which is:

(a) Parallel to L (b) Perpendicular to L

4.(i) A line passes through the points A(4, -4) and B(2, 6). Find the equation of this line.

(ii) A line makes an angle of 30^0 with the positive direction of the x-axis, as shown in the diagram. Find the equation of this line.



(iii) A line, L, has equation $y = -\frac{3}{4}x - 2$.

Write down the equation of a line, passing through (-2, 4), which is:

(a) Parallel to L (b) Perpendicular to L

Applications Assessment Standard 1.1 Answers

1(i) $m = -\frac{1}{2}$, $y - 2 = -\frac{1}{2}(x + 6)$

(ii) $m = \tan 40^0 = 0.84$ (to 2 d.p.)

(iii) (a) $y - 1 = 4(x - 2)$ (b) $y - 1 = -\frac{1}{4}(x - 2)$

2(i) $m = 2$, $y - 1 = 2(x - 6)$

(ii) $m = \tan 50^0 = 1.19$ (to 2 d.p.)

(iii) (a) $y - 2 = -3(x - 4)$ (b) $y - 2 = \frac{1}{3}(x - 4)$

3(i) $m = \frac{1}{3}$, $y - 2 = \frac{1}{3}(x + 4)$

(ii) $m = \tan 75^0 = 3.73$ (to 2 d.p.)

(iii) (a) $y + 1 = \frac{1}{4}(x - 3)$ (b) $y + 1 = -4(x - 3)$

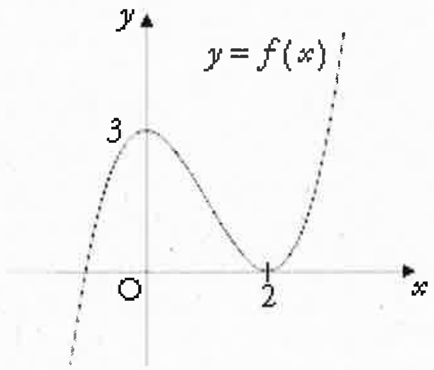
4(i) $m = -5$, $y - 6 = -5(x - 2)$

(ii) $m = \tan 30^0 = 0.58$ (to 2 d.p.)

(iii) (a) $y - 4 = -\frac{3}{4}(x + 2)$ (b) $y - 4 = \frac{4}{3}(x + 2)$

Expressions and Functions Assessment Standard 1.3

1. The diagram below shows part of the graph of y = f(x).



(a) Sketch the graph of y = -f(x).

(b) On a separate diagram, sketch the graph of y = f(x + 4).

2. Diagrams 1 and 2 below show part of the graph of y = f(x).

(a) On Diagram 1, draw the graph of y = -f(x).

(b) On Diagram 2, draw the graph of y = f(x + 4)

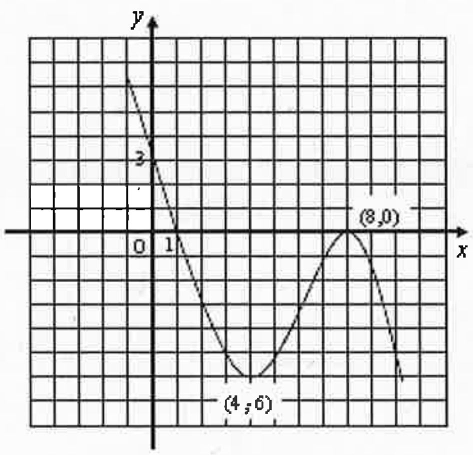


Diagram 1

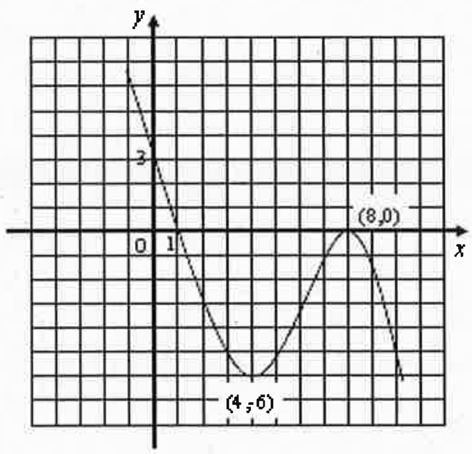


Diagram 2

3. Diagrams 1 and 2 below show part of the graph of y = f(x).

(a) On Diagram 1, draw the graph of y = -f(x).

(b) On Diagram 2, draw the graph of y = f(x - 4).

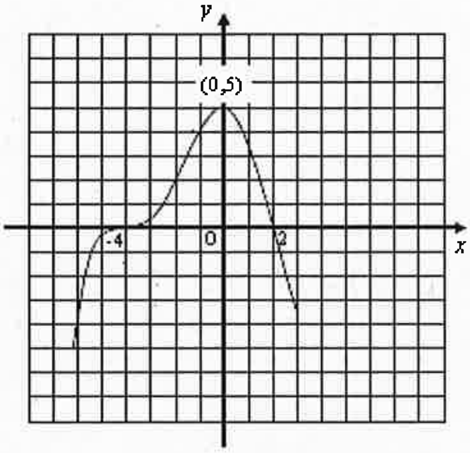


Diagram 1

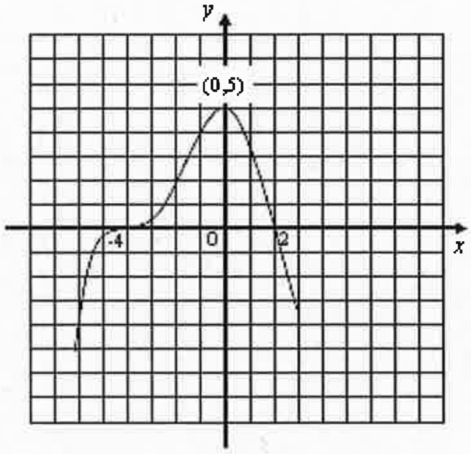


Diagram 2

4. Diagrams 1 and 2 below show part of the graph of y = f(x).

(a) On Diagram 1, draw the graph of y = -f(x) + 2.

(b) On Diagram 2, draw the graph of y = f(x - 3).

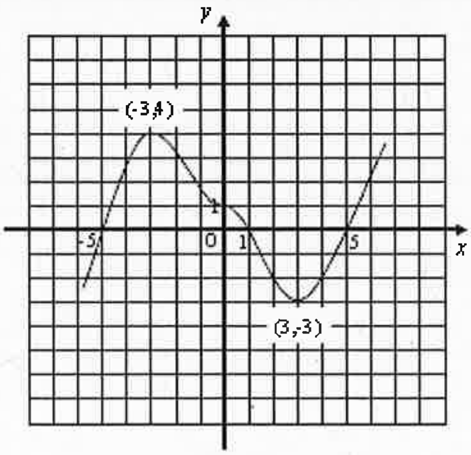


Diagram 1

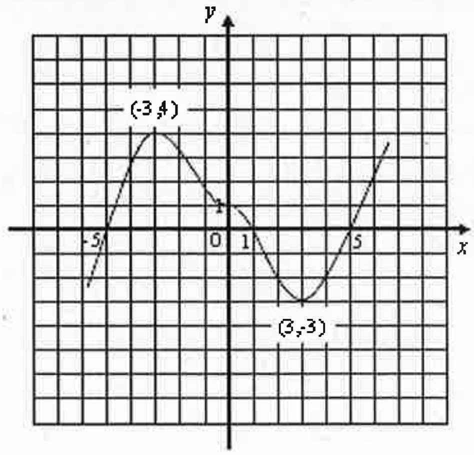
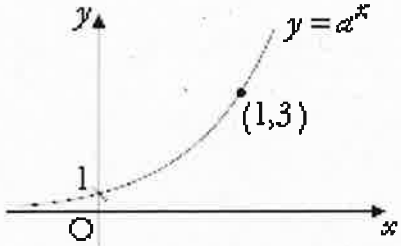


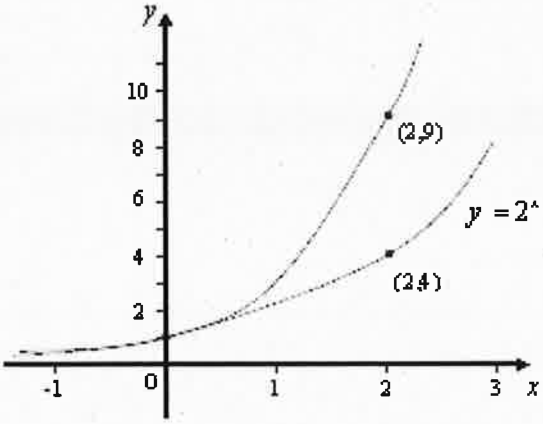
Diagram 2

5. The curve $y = a^x$ is shown in the diagram below.

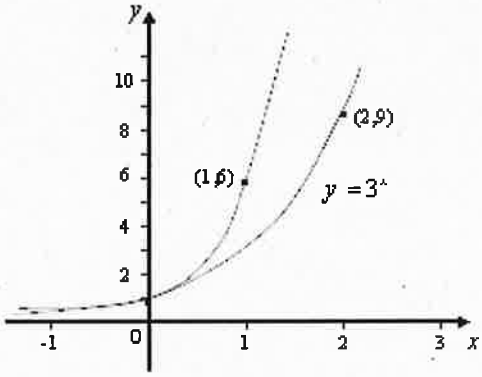


Given that the curve passes through the point (1, 3), write down the value of a.

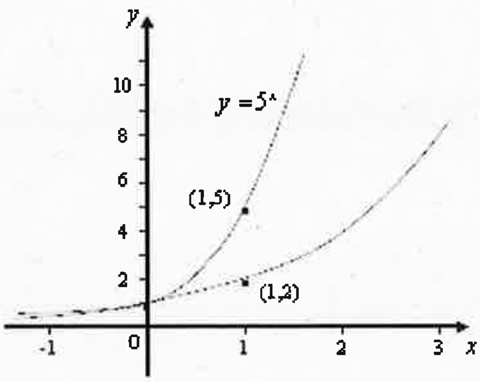
6. The graph of $y = 2^x$ is shown in the diagram below. Write down the equation of the graph of the exponential function of the form $y = a^x$ which passes through the point (2, 9) as shown.



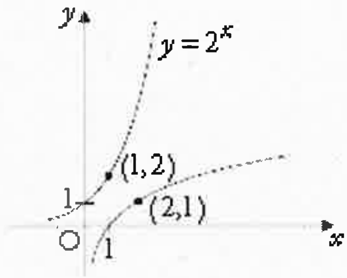
7. The graph of $y = 3^x$ is shown in the diagram below. Write down the equation of the graph of the exponential function of the form $y = a^x$ which passes through the point (1, 6) as shown.



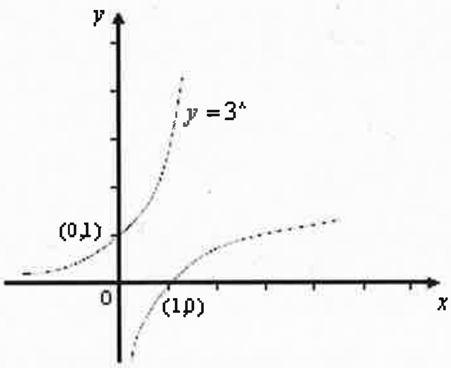
8. The graph of $y = 5^x$ is shown in the diagram below. Write down the equation of the graph of the exponential function of the form $y = a^x$ which passes through the point (1, 2) as shown.



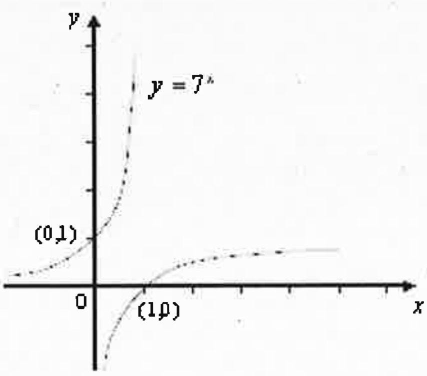
9. The diagram below shows the graph of the function $f(x) = 2^x$ and its inverse function. Write down the formula for the inverse function.



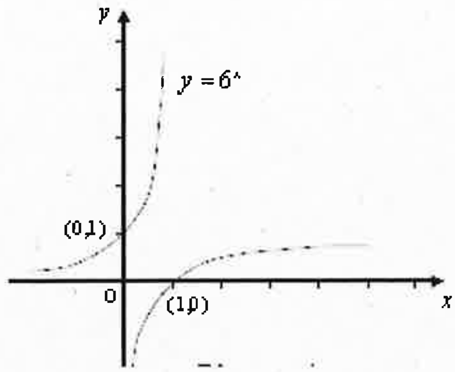
10. The diagram below shows the graph of the function $f(x) = 3^x$ and its inverse function. Write down the formula for the inverse function.



11. The diagram below shows the graph of the function $f(x) = 7^x$ and its inverse function. Write down the formula for the inverse function.



12. The diagram below shows the graph of the function $f(x) = 6^x$ and its inverse function. Write down the formula for the inverse function.



13. (a) Two functions f and g are defined by $f(x) = x^3$ and $g(x) = 2x - 4$. Find an expression for $f(g(x))$.

(b) Functions h and k are defined on suitable domains by $h(x) = 5x$ and $k(x) = \tan x$. Find an expression for $k(h(x))$.

14.(a) Two functions f and g are defined by $f(x) = x^2 - 1$ and $g(x) = 3x - 1$. Find an expression for $f(g(x))$.

(b) Functions h and k are defined on suitable domains by $h(x) = 4x$ and $k(x) = \cos x$. Find an expression for $k(h(x))$.

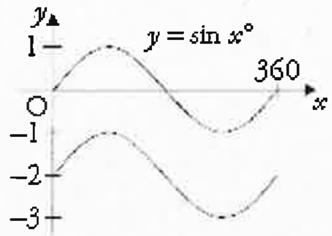
15.(a) Two functions f and g are defined by $f(x) = 2x^2$ and $g(x) = x + 1$. Find an expression for $f(g(x))$.

(b) Functions h and k are defined on suitable domains by $h(x) = \sin x$ and $k(x) = \frac{1}{2}x$. Find an expression for $k(h(x))$.

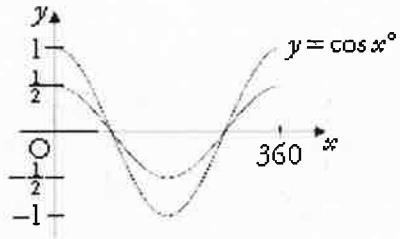
16. (a) Two functions f and g are defined by $f(x) = x^2 + x$ and $g(x) = 3x + 1$. Find an expression for $f(g(x))$.

(b) Functions h and k are defined on suitable domains by $k(x) = \cos x$ and $h(x) = (2x + \pi)$. Find an expression for $k(h(x))$.

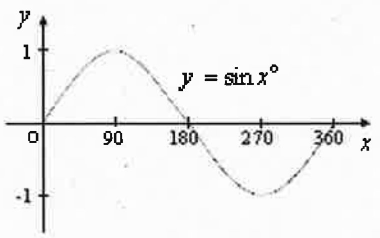
17.(a) The diagram below shows the curve y = sin x^o and a related curve. Write down the equation of the related curve.

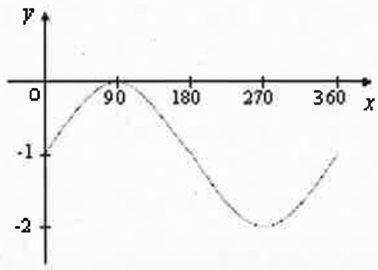


(b) The diagram below show the curve y = cos x^o and a related curve. Write down the equation of the related curve.

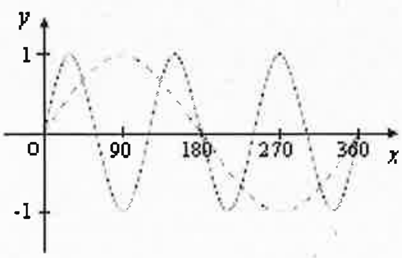


18.(a) The diagrams below show part of the graph of y = sin x^o and the graph of a related function. Write down the equation of the related function.

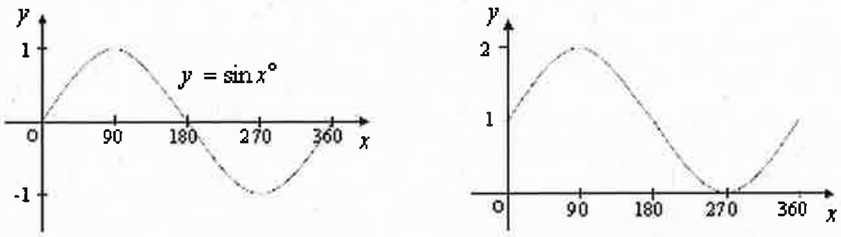




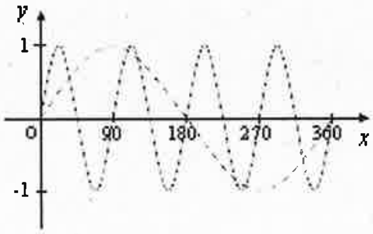
(b) The diagram below shows part of the graph of y = sin x^0 (drawn as a broken line) and the graph of a related function. Write down the equation of the related function.



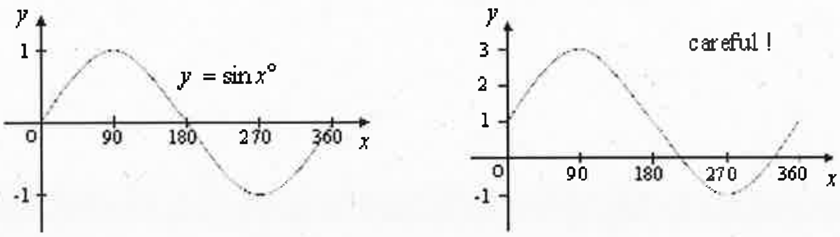
19.(a) The diagrams below show part of the graph of y = sin x^0 and the graph of a related function. Write down the equation of the related function.



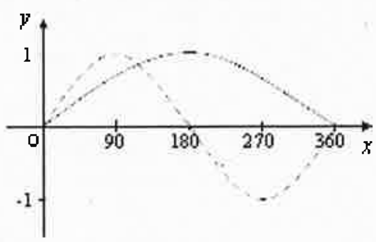
(b) The diagram below shows part of the graph of y = sin x^0 (drawn as a broken line) and the graph of a related function. Write down the equation of the related function.



20.(a) The diagrams below show part of the graph of y = sin x^0 and the graph of a related function. Write down the equation of the related function.



(b) The diagram below shows part of the graph of y = sin x^0 (drawn as a broken line) and the graph of a related function. Write down the equation of the related function.



21. The functions f and g, defined on suitable domains, are given by

$f(x) = 2x - 4$ and $g(x) = \frac{1}{2}x + 2$.

A third function $h(x)$ is defined as $h(x) = g(f(x))$.
Find an expression for $h(x)$ and state the relationship between $f(x)$ and $g(x)$.

22. The functions f and g, defined on suitable domains, are given by

$f(x) = 3x + 9$ and $g(x) = \frac{1}{3}x - 3$.

Find an expression for $h(x) = g(f(x))$ and state the relationship between $f(x)$ and $g(x)$.

23. The functions f and g, defined on suitable domains, are given by $f(x) = 2x - 1$ and $g(x) = \sqrt{x}$. Find an expression for $g(f(x))$ and state a suitable domain.

24. The functions f and g, defined on suitable domains, are given by

$f(x) = \frac{1}{x^2-4}$ and $g(x) = 2x + 1$.

Find an expression for $h(x) = g(f(x))$ and state a suitable domain.

25. For $0 \le x \le \frac{\pi}{4}$ a curve with equation $y = a\cos bx + c$ has a single maximum value at $(0, 1)$ and a single minimum value at $(\frac{\pi}{4}, -5)$.

(a) Write down the values of a, b and c.

(b) Sketch the curve given by the equation in part (a).

26. For $0 \le x \le \frac{\pi}{2}$ a curve with equation $y = a\sin bx + c$ has a single maximum value at $(\frac{\pi}{8}, 7)$ and a single minimum value at $(\frac{3\pi}{8}, 3)$.

(a) Write down the values of a, b and c.

(b) Sketch the curve given by the equation in part (a).

Expressions and Functions Assessment Standard 1.3 Answers

1. (a) Reflection in x-axis (b) Translation 4 units left

2. (a) Reflection in x-axis (b) Translation 4 units left

3. (a) Reflection in x-axis (b) Translation 4 units right

4. (a) Reflection in x-axis then up 2 (b) Translation 3 units right

5. $a = 3$

6. $y = 3^x$

7. $y = 6^x$

8. $y = 2^x$

9. $y = \log_2 x$

10. $y = \log_3 x$

11. $y = \log_7 x$

12. $y = \log_6 x$

13.(a) $f(g(x))) = (2x - 4)^3$ (b) $k(h(x)) = \tan 5x$

14.(a) $f(g(x))) = (3x - 1)^2 - 1 = 9x^2 - 6x$ (b) $k(h(x)) = \cos 4x$

15.(a) $f(g(x))) = 2(x + 1)^2$ (b) $k(h(x)) = \frac{1}{2}\sin x$

16.(a) $f(g(x))) = (3x + 1)^2 + 3x + 1 = 9x^2 + 9x + 2$ (b) $k(h(x)) = \tan 5x$

17.(a) $y = \sin x^0 - 2$ (b) $y = \frac{1}{2}\cos x^0$

18. (a) $y = \sin x^0 - 1$ (b) $y = \sin 3x^0$

19. (a) $y = \sin x^0 + 1$ (b) $y = \sin 4x^0$

20. (a) $y = 2\sin x^0 + 1$ (b) $y = \sin \frac{1}{2}x^0$

21. $h(x) = x$. $f(x)$ and $g(x)$ are inverse functions.

22. *h(x)* = x. *f*(*x*) and *g*(*x*) are inverse functions.

23. *h(x)* = $\sqrt{2x-1}$. Domain: $2x - 1 \geq 0$, i.e. $x \in R: x \geq \frac{1}{2}$

24. *h(x)* = $2(\frac{1}{x^2-4}) + 1$. Domain: $x^2 \neq 4$, i.e. $x \in R: x \neq 2$ or -2

25. $y = 3\cos 4x - 2$ so $a = 3$, $b = 4$ and $c = -2$.

26. $y = 2\sin 4x + 5$ so $a = 2$, $b = 4$ and $c = 5$.

Relationships and Calculus Assessment Standard 1.3

1. Given that $y = \frac{x^5-3}{x^3}$ for $x \neq 0$, find $\frac{dy}{dx}$.

2. Given that $y = \frac{x^4+2}{x^3}$ for $x \neq 0$, find $\frac{dy}{dx}$.

3. Given that $y = \frac{1+x^4}{x^2}$ for $x \neq 0$, find $\frac{dy}{dx}$.

4. Given that $y = \frac{3+x^6}{x^4}$ for $x \neq 0$, find $\frac{dy}{dx}$.

5. Given $y = \frac{6}{x^5} + 3x^{1/2}$, $x \neq 0$, find $\frac{dy}{dx}$.

6. Given $y = \frac{3}{x^4} + 2x^{1/5}$, $x \neq 0$, find $\frac{dy}{dx}$.

7. Given $y = \frac{2}{x^3} + 8x^{1/4}$, $x \neq 0$, find $\frac{dy}{dx}$.

8. Given $y = \frac{5}{x^2} + 6x^{1/3}$, $x \neq 0$, find $\frac{dy}{dx}$.

9. Given $y = 3\sin x$, find $\frac{dy}{dx}$.

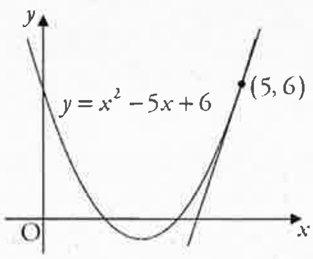
10. Given $y = \frac{3}{4}\cos x$, find $\frac{dy}{dx}$.

11. Differentiate $5\sin x$ with respect to x.

12. Differentiate $8\cos x$ with respect to x.

13. A sketch of the curve with equation $y = x^2 - 5x + 6$ is shown in the diagram. A tangent has been drawn at the point P(5, 6).

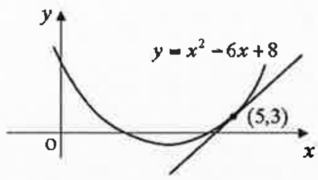
    Find the equation of the tangent at P.



14. A sketch of the curve with equation $y = x^2 - 6x + 8$ is shown in the diagram. A tangent has been drawn at the point P(5, 3).

(a) Find the equation of the tangent at P.

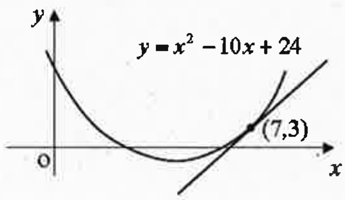
(b) Explain why a tangent to the curve at (3, -1) is parallel to the x-axis.



15. A sketch of the curve with equation $y = x^2 - 10x + 24$ is shown in the diagram. A tangent has been drawn at the point P(7, 3).

(a) Find the equation of the tangent at P.

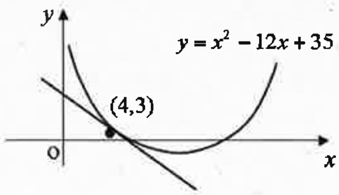
(b) Explain why a tangent to the curve at (5, -1) is parallel to the x-axis.



16. A sketch of the curve with equation $y = x^2 - 12x + 35$ is shown in the diagram. A tangent has been drawn at the point P(4, 3).

(a) Find the equation of the tangent at P.

(b) Explain why a tangent to the curve at (6, -1) is parallel to the x-axis.



17. Find the equation of the tangent to $y = 3x^2$ at $x = 2$.

18. Find the equation of the tangent to $y = x^2 + 2x$ at $x = 1$. What can you say about the tangent line at $x = -1$?

19. At time t seconds a ball thrown vertically upwards has height $h(t) = 2 + 10t - 5t^2$ metres.
(a) What is its initial height?
(b) When does it return to its initial height?
(c) Calculate its speed at $t = 1$. Explain.
(d) Find its initial speed, and its speed when it returns to its starting point.

20. Doug's model of the "Forties Flier" runs along a straight track. Its displacement OP metres from the signal at O after t seconds is given by $x(t) = 1 + 4t - t^2$, $t \geq 0$.
(a) Find an expression for its velocity at time t.
(b) At what time is the velocity zero? What does this mean?
(c) Calculate the velocity at $t = 3$. What does the negative sign tell you?

Relationships and Calculus Assessment Standard 1.3 Answers

1. $\frac{dy}{dx} = 2x + 9x^{-4}$

2. $\frac{dy}{dx} = 1 - 6x^{-4}$

3. $\frac{dy}{dx} = -2x^{-3} + 2x$

4. $\frac{dy}{dx} = -12x^{-5} + 2x$

5. $\frac{dy}{dx} = \frac{-30}{x^6} + \frac{3}{2}x^{-1/2}$

6. $\frac{dy}{dx} = \frac{-12}{x^5} + \frac{2}{5}x^{-4/5}$

7. $\frac{dy}{dx} = \frac{-6}{x^4} + 2x^{-3/4}$

8. $\frac{dy}{dx} = \frac{-10}{x^3} + 2x^{-2/3}$

9. $\frac{dy}{dx} = 3\cos x$

10. $\frac{dy}{dx} = -\frac{3}{4}\sin x$

11. $\frac{dy}{dx} = 5\cos x$

12. $\frac{dy}{dx} = -8\sin x$

13. $y - 6 = 5(x - 5)$

14. (a) $y - 3 = 4(x - 5)$ (b) At $x = 3$, $m = 0$

15. (a) $y - 3 = 4(x - 7)$ (b) At $x = 5$, $m = 0$

16. (a) $y - 3 = -4(x - 4)$ (b) At $x = 6$, $m = 0$

17. $y = 12x - 12$

18. $y = 4x - 1$
At $x = -1$, gradient = 0 and the tangent line is horizontal. $\frac{dy}{dx} = \frac{-6}{x^4} + 2x^{-3/4}$

19. (a) $h = 2$

(b) $t = 2$
(c) Speed = 0. The ball is at its highest point and stationary.
(d) Initial speed = 10. Final speed = -10.

20. (a) $v = \frac{dx}{dt} = 4 - 2t$
(b) At $t = 2$, the train has stopped.
(c) $v = -2$. The train is reversing.

Applications Assessment Standard 1.3

1. A pond is treated weekly with a chemical to ensure that the number of bacteria is kept low. It is estimated that the chemical kills 68% of all bacteria. Between the weekly treatments, it is estimated that 600 million new bacteria appear. There are u_n million bacteria at the start of a particular week.

(a) Write down a recurrence relation for u_{n+1}, the number of millions of bacteria at the start of the next week.

(b) It is known that the pond cannot sustain more than 900 million bacteria at any one time. In the long term, can the pond sustain the number of bacteria, given the above conditions?

2. In a small rabbit colony one eighth of the existing rabbits are eaten by predators each summer. However, during the winter 24 rabbits are born. There are u_n rabbits at the start of a particular summer.

(a) Write down a recurrence relation for u_{n+1}, the number of rabbits at the start of the next summer.

(b) It is known that the colony cannot sustain more than 180 rabbits at any one time. In the long term, can the colony sustain the number of rabbits?

3. In a small colony 20% of the existing insects are eaten by predators each day. However, during the night 400 insects are hatched. There are u_n insects at the start of a particular day.

(a) Write down a recurrence relation for u_{n+1}, the number of insects at the start of the next day.

(b) Find the limit of the sequence generated by this recurrence relation and explain what the limit means in the context of this question.

4. For an established ant hill 6% of the worker ants fail to return at the end of each day. However, during the night 540 worker ants are hatched. There are u_n worker ants at the start of a particular day.

(a) Write down a recurrence relation for u_{n+1}, the number of worker ants at the start of the next day.

(b) Find the limit of the sequence generated by this recurrence relation and explain what the limit means in the context of this question.

Applications Assessment Standard 1.3 Answers

1. (a) $u_{n+1} = 0.32\, u_n + 600$

(b) A limit exists since $-1 < 0.32 < 1$ and L = 882.35 (to 2d.p.). In the long term the pond will be able to sustain the number of bacteria since 882.35 < 900.

2. (a) $u_{n+1} = \frac{7}{8} u_n + 24$

(b) A limit exists since $-1 < \frac{7}{8} < 1$ and L = 192. In the long term the colony will not be able to sustain the number of rabbits since 192 > 180.

3. (a) $u_{n+1} = 0.8\, u_n + 400$

(b) A limit exists since $-1 < 0.8 < 1$ and L = 2000. In the long term the population of the insects will settle to 2000.

4. (a) $u_{n+1} = 0.94\, u_n + 540$

(b) A limit exists since $-1 < 0.94 < 1$ and L = 9000. In the long term the population of the worker ants will settle to 9000.