Chapter 16

Exercise 16A

1 a The greatest value of $f(x)$ is $\frac{-131}{27}$ and the least value of $f(x)$ is –23

b The greatest value of $g(x)$ is 30 and the least value of $g(x)$ is –70

c The greatest value of $h(x)$ is –8 and the least value of $h(x)$ is –18.4

d The greatest value of y is 9 and the least value of y is –266

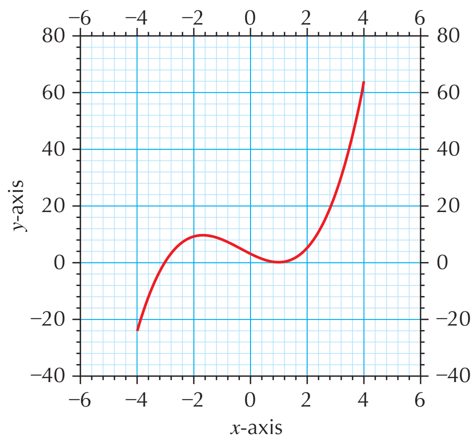
e The greatest value of y is 75 and the least value of y is –5

f The greatest value of y is 128 and the least value of y is –1.69

2 a The curve crosses the x-axis at (1,0) and at (–3,0), the y-axis at (0,3)

b Minimum SP at (1,0), Maximum SP at $\left(-\frac{5}{3}, 9\frac{12}{25}\right)$

c i



ii The greatest value of $f(x)$ is 5 and the least value of $f(x)$ is 0

3 a Pupil's own answer

b The greatest value of $h(x)$ is 16 and the least value of $h(x)$ is –92

4 a $f(x) = 3(x-1)^2 + 5$

b Yes, because $f'(x) > 0\ \forall x$, hence $f(x)$ is always rising and the greatest and least values will occur at the endpoints of the interval

5 a i, ii Minimum SP at $\left(\frac{4\pi}{3}, -2\sqrt{3}\right)$, Maximum SP at $\left(\frac{\pi}{3}, 2\sqrt{3}\right)$

b The greatest value of $f(x)$ is 3 and the least value of $f(x)$ is $-2\sqrt{3}$

c Hint: use trigonometric addition formulas

d Hint: consider how the cosine varies on the given interval

Challenge

a Hint: the function is the product of two squared values

b Hint: if $x = k$ is an axis of symmetry, $f(x + k)$ will be an even function

c Hint: if you consider the even function, the minimum difference and hence the maximum value will be when $x = 0$

Exercise 16B

1 The maximum value of Claire's shares is £28,000 after 20 days, the minimum value is £2,000 at the beginning.

2 a $b = 30 - x$

b Hint: $A = b \times l$

c Maximum area $A = 225\,cm^2$.

3 a Hint: the height is x, find expressions for the length and breadth in terms of x

b The maximum volume is $V = 90.74\ inch^2$ when $x = \frac{5}{3}\ inch$.

4 a Hint: use the volume to find h in terms of x

b $x = 15cm^2$

c Minimum surface area $S = 1350\,cm^2$

5 a i Hint: consider only prices between £1 and £6

ii Since the demand for your app is linear, it can be represented by a straight line, where you have costs on the x-axis and sales on the y-axis

b Hint: Profit = sales × price – costs

c $x = £3$, Maximum Profit = £114,000

d Pupil's own answer

6 a Hint: use the volume of the cylinder to find h in terms of r.

b $r = 3.25m$, Minimum Surface $= 2808m^2$

7 a Hint: use the volume of the cylinder to find h in terms of r.

b $r = \sqrt[3]{\frac{9}{20\pi}}\,cm$, Minimum Surface $= 827.37cm^2$

8 a Hint: use the surface area to find h in terms of x

b $x = \sqrt{\frac{A}{2}}\,cm,\ h = \sqrt{\frac{A}{2}}\,cm$

9 Max area = 7.56 square units

10 a $h = \frac{1000}{\sqrt{3}x^2}\,cm$

b Area of cross-section

$$= \frac{1}{2} \bullet \frac{x^2\sqrt{3}}{2} = \frac{x^2\sqrt{3}}{4}$$

Total SA $= 2 \times \Delta + 3 \times \square$

$$= \frac{2 \bullet x^2\sqrt{3}}{4} + 3hx$$

$$V = 250 \quad and \quad V = \frac{hx^2\sqrt{3}}{4}$$

$$\therefore \frac{hx^2\sqrt{3}}{4} = 250 \Rightarrow h = \frac{1000}{x^2\sqrt{3}}$$

$$SA = \frac{x^2\sqrt{3}}{2} + 3 \bullet \frac{1000}{x^2\sqrt{3}} \bullet x$$

$$= \frac{x^2\sqrt{3}}{2} + \frac{3\sqrt{3} \bullet 1000 \bullet x}{3x^2}$$

$$= \frac{x^2\sqrt{3}}{2} + \frac{1000 \bullet x \bullet \sqrt{3}}{x^2}$$

$$= \frac{x^2\sqrt{3}}{2} + \frac{1000 \bullet x \bullet \sqrt{3} \bullet 2}{2x}$$

$$= \frac{x^2 \bullet \sqrt{3}}{2} + \frac{2000}{x} \bullet \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{2}\left(x^2 + \frac{2000}{x}\right)$$

c $x = 10cm$, minimum amount of plastic $= 150\sqrt{3}\,cm^2$

11 a $P = 1400x^3 - 3410x^2 + 2100x$

b Maximum profit = 384.28 £/tank

12 a $x = 60t,\ y = 80t$

b Hint: d is the hypotenuse of a rectangle where one side is x and the other one is $40 - y$

c $9.6\,min$

13 $6\,min$

14 a Hint: use Pythagoras'theorem to find the third side of the triangle

b Maximum perimeter $= 30\left(1 + \sqrt{2}\right)cm$

c Pupil's own answer

d $\alpha = \frac{\pi}{4}$

15 $P\left(\pm\frac{\sqrt{2}}{2}, \frac{3}{2}\right)$

16 Maximum Area $= 2r^2$

Challenge 1

$$\frac{C}{V} = \frac{4}{9}$$

Challenge 2

$$A_{triangle} = \frac{9k^2\sqrt{3}}{\left(18+\sqrt{3}\pi\right)^2},\ A_{circle} = \frac{3\pi k^2}{4\left(18+\sqrt{3}\pi\right)^2}$$

Exercise 16C

1 a $v = 4t$

b $v(2) = 8m$

c $a = 4$

2 a $s(0) = 4m$

b $v(4) = 11\,ms^{-1}$

c $t_1 = 0.33s,\ t_2 = 3s$

d $s(3) = 0$, the particle is at the origin of the x-axis.

e $v = -5\,ms^{-1}$. v is negative, which means the particle is going back with respect to the previous direction.

3 **a** $a(4) = 1.33ms^{-2}$

b $t = 50.6s$

4 The radius is $6m$ when time is $t = 3s$. $A'(3) = 36\pi$

5 $\frac{dr}{dt} = \frac{75}{128\pi} cms^{-1}$

6 **a** Hint: height and radius are proportional while the volume is changing

b $\frac{dh}{dt} = \frac{50}{9\pi} cms^{-1}$

7 **a** Pupil's own answer

b Hint: $1l = 1dm^3$

c **i** Hint: $g(x) = y$, $f(x) = [g(x)]^2$

ii $3y^2 \frac{dy}{dx}$

d Pupil's own answer

e Hint: $\frac{dr}{dt} = \frac{2}{25\pi} ms^{-1}$, then convert seconds in hours and write the answer in decimal number.

f 19 *hours* and 27 *minutes*.

$\frac{dV}{dt} = 0$ and $\frac{dr}{dt} = 0$, because there is no more variation either in volume (the tank is empty now) or in radius (if we don't consider winds and tides).